

# An Useful Model for Charge Pump Converter

Hanns Chiu

## Introduction:

This application note focuses on the analysis of the charge pump and its equivalent circuit. The AIC1845 is a micro-power charge pump DC/DC converter that produces a regulated 5V output. This kind of converter uses capacitors to store and transfer energy. Since the capacitors can not change their voltage level abruptly, the voltage ratio of  $V_{OUT}$  over  $V_{IN}$  is limited to some range. Capacitive voltage conversion is obtained by switching a capacitor periodically. It first charges the capacitor by connecting it across a voltage source and then connects it to the output. Referring to Fig.1, During the on state of internal clock,  $Q_1$  and  $Q_4$  are closed, which charges  $C_1$  to  $V_{IN}$  level. During the off state,  $Q_3$  and  $Q_2$  are closed. The output voltage is  $V_{IN}$  plus  $V_{C1}$ , that is,  $2 V_{IN}$ .

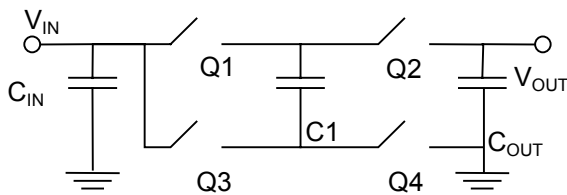


Fig.1 The circuit of charge pump

## Analysis:

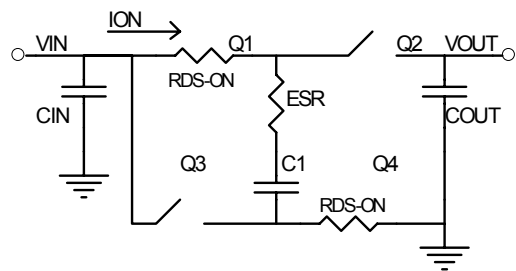


Fig.2 The on state of charge pump circuit

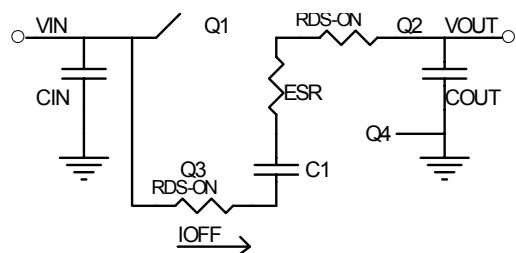


Fig.3 The off state of charge pump circuit

Referring to Fig.2 and Fig.3, here shows the circuit of charge pump at different states of operation.  $R_{DS-ON}$  is the resistance of the switching element at conduction. ESR is the equivalent series resistance of the flying capacitor  $C_1$ .  $I_{ON-AVE}$  and  $I_{OFF-AVE}$  are the average current during on state and off state, respectively. D is the duty cycle, which means the proportion the on state takes. Let's take advantage of conservation of charge for capacitor  $C_1$ . Assume that the capacitor  $C_1$  has reached its steady state. The amount of charge flowing into  $C_1$  during on state is equal to that flowing out of  $C_1$  at off state.

$$I_{ON-AVE} \times DT = I_{OFF-AVE} \times (1-D)T \dots\dots\dots(1)$$

$$I_{ON-AVE} \times D = I_{OFF-AVE} \times (1-D) \dots\dots\dots(2)$$

$$\begin{aligned}
 I_{IN} &= I_{ON-AVE} \times D + I_{OFF-AVE} \times (1-D) \\
 &= 2 \times I_{ON-AVE} \times D \quad \dots\dots (3) \\
 &= 2 \times I_{OFF-AVE} \times (1-D)
 \end{aligned}$$

$$\begin{aligned}
 I_{OUT} &= I_{OFF-AVE} \times (1-D) \\
 I_{IN} &= 2I_{OUT} \dots\dots\dots (4)
 \end{aligned}$$

For AIC1845, the controller takes the PSM (Pulse Skipping Modulation) control strategy. When the duty cycle is limited to 0.5, there will be:

$$\begin{aligned}
 I_{ON-AVE} \times 0.5 \times T &= I_{OFF-AVE} \times (1-0.5) \times T \\
 I_{ON-AVE} &= I_{OFF-AVE} \dots\dots\dots (5)
 \end{aligned}$$

According to the equation (4), we know that as long as the flying capacitor C1 is at steady state, the input current is twice the output current. The efficiency of charge pump is given below:

$$\eta = \frac{V_{OUT} \times I_{OUT}}{V_{IN} \times I_{IN}} = \frac{V_{OUT} \times I_{OUT}}{V_{IN} \times 2I_{OUT}} = \frac{V_{OUT}}{2V_{IN}} \dots\dots (6)$$

Let's consider the power dissipation of  $R_{DS-ON}$  and ESR. Assume that the  $R_{DS-ON}$  of each switching element is equal. The approximation of the power loss of  $R_{DS-ON}$  and ESR are given below:

$$\begin{aligned}
 P_{R_{DS-ON}} &\cong I_{ON-AVE}^2 \times 2R_{DS-ON} \times D + I_{OFF-AVE}^2 \times 2R_{DS-ON} \times (1-D) \\
 &= \left(\frac{I_{IN}}{2D}\right)^2 \times 2R_{DS-ON} \times D + \left(\frac{I_{OUT}}{1-D}\right)^2 \times 2R_{DS-ON} \times (1-D) \\
 &= \left(\frac{2I_{OUT}}{2D}\right)^2 \times 2R_{DS-ON} \times D + \left(\frac{I_{OUT}}{1-D}\right)^2 \times 2R_{DS-ON} \times (1-D) \\
 &= I_{OUT}^2 \times \left(\frac{2}{D} R_{DS-ON}\right) + I_{OUT}^2 \times \left(\frac{2}{1-D} R_{DS-ON}\right) \\
 &= I_{OUT}^2 \times \frac{2}{D(1-D)} \times R_{DS-ON} \\
 &\dots\dots\dots (7)
 \end{aligned}$$

$$\begin{aligned}
 P_{ESR} &\cong I_{ON-AVE}^2 \times ESR \times D + I_{OFF-AVE}^2 \times ESR \times (1-D) \\
 &= \left(\frac{I_{IN}}{2D}\right)^2 \times ESR \times D + \left(\frac{I_{OUT}}{1-D}\right)^2 \times ESR \times (1-D) \\
 &= I_{OUT}^2 \times ESR \times \frac{1}{D} + I_{OUT}^2 \times ESR \times \frac{1}{1-D} \\
 &= I_{OUT}^2 \times ESR \times \frac{1}{D(1-D)}
 \end{aligned}$$

When the duty cycle is 0.5, the power loss of switching element is

$$\begin{aligned}
 P_{R_{DS-ON}} &\cong I_{OUT}^2 \times \frac{2}{0.5(1-0.5)} \times R_{DS-ON} \\
 &= I_{OUT}^2 \times 8R_{DS-ON} \quad \dots\dots\dots (8)
 \end{aligned}$$

$$\begin{aligned}
 P_{ESR} &\cong I_{OUT}^2 \times ESR \times \frac{1}{0.5(1-0.5)} \\
 &= I_{OUT}^2 \times 4ESR
 \end{aligned}$$

In fact, no matter the current is at on state or off state, it decays exponentially rather than flows steadily. And the root mean square value of exponential decay is not equal to that of steady flow. That is why the approximation comes from.

Let's treat the charge pump circuit in another approach and lay the focus on the flying capacitor C<sub>1</sub>. Referring to Fig.2, when the circuit is at the on state, the voltage across C<sub>1</sub> is:

$$V_{C-ON}(t) = V_{IN} - 2R_{DS-ON} \times I_{ON}(t) - (ESR \times I_{ON}(t)) \dots (9)$$

The average of  $V_{C1}$  during the on state is:

$$V_{C-ON-AVE} = V_{IN} - (2R_{DS-ON} \times I_{ON-AVE}) - (ESR \times I_{ON-AVE}) \dots (10)$$

Similarly, referring to Fig.3, when the circuit is at the off state, the voltage of C1 is:

$$V_{C-OFF}(t) = V_{OUT} - V_{IN} + 2R_{DS-ON} \times I_{OFF}(t) + ESR \times I_{OFF}(t) \dots (11)$$

The average of  $V_{C1}$  during the off state is:

$$V_{C-OFF-AVE} = V_{OUT} - V_{IN} + 2R_{DS-ON} \times I_{OFF-AVE} + ESR \times I_{OFF-AVE} \dots (12)$$

The difference of charge stored in  $C_1$  between on state and off state is the net charge transferred to the output in one cycle.

$$\begin{aligned} \Delta Q &= Q_{ON} - Q_{OFF} \\ &= C_1 \times (V_{C1-ON-AVE} - V_{C1-OFF-AVE}) \\ &= C_1 \times (2V_{IN} - V_{OUT} - 2R_{DS-ON} \times I_{ON-AVE} - 2R_{DS-ON} \times I_{OFF-AVE} - ESR \times I_{ON-AVE} - ESR \times I_{OFF-AVE}) \dots (13) \\ &= C_1 \times (2V_{IN} - V_{OUT} - 2R_{DS-ON} \times \frac{I_{OUT}}{D} - 2R_{DS-ON} \times \frac{I_{OUT}}{1-D} - ESR \times \frac{I_{OUT}}{D} - ESR \times \frac{I_{OUT}}{1-D}) \\ &= C_1 \times [2V_{IN} - V_{OUT} - (2R_{DS-ON} + ESR) \times I_{OUT} \times \frac{1}{D(1-D)}] \end{aligned}$$

Thus the output current can be written as

$$\begin{aligned} I_{OUT} &= f \times \Delta Q = f \times (Q_{ON} - Q_{OFF}) \\ &= f \times C_1 \times [2V_{IN} - V_{OUT} - (2R_{DS-ON} + ESR) \times I_{OUT} \times \frac{1}{D(1-D)}] \dots (14) \end{aligned}$$

When the duty cycle is 0.5, the output current can be

$$\begin{aligned} I_{OUT} &= f \times C_1 \times [2V_{IN} - V_{OUT} - (2R_{DS-ON} + ESR) \times I_{OUT} \times \frac{1}{0.5(1-0.5)}] \dots (15) \\ &= fC_1 \times [2V_{IN} - V_{OUT} - (8R_{DS-ON} + 4ESR) \times I_{OUT}] \end{aligned}$$

And equation (15) can be re-written as:

$$2V_{IN} - V_{OUT} = \frac{1}{fC_1} \times I_{OUT} + (8R_{DS-ON} + 4ESR) \times I_{OUT} \dots (16)$$

According to the equation (16), when the duty cycle is 0.5, the equivalent circuit of the charge pump is shown in Fig.4. The term  $8 R_{DS-ON}$  is the total effect of switching resistance,  $1/fC_1$  is the effect of the flying capacitor and  $4ESR$  is its equivalent resistance.

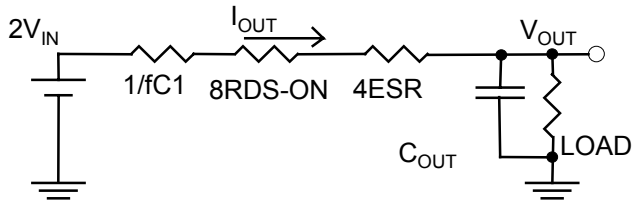


Fig.4 The equivalent circuit of charge pump

to make the duty cycle of AIC1580 to be 0.5. The experiment data and the corresponding  $R_{DS-ON}$  (evaluated from the equivalent circuit in Fig.4) are listed in Table.1. The value of  $R_{DS-ON}$  is then assigned to all switching resistors of the simulation circuit, Fig.6. For the given loading  $R_{LOAD}$ , the output voltage and output current of the experiment results and those from simulation are shown in Fig.7. From Fig.7, it is easily seen that the derived equivalent circuit in Fig.4 is quite corresponding to the original charge pump circuit.

**Experimental and Simulation Results:**

For the reason of simplicity, the output loading is chosen

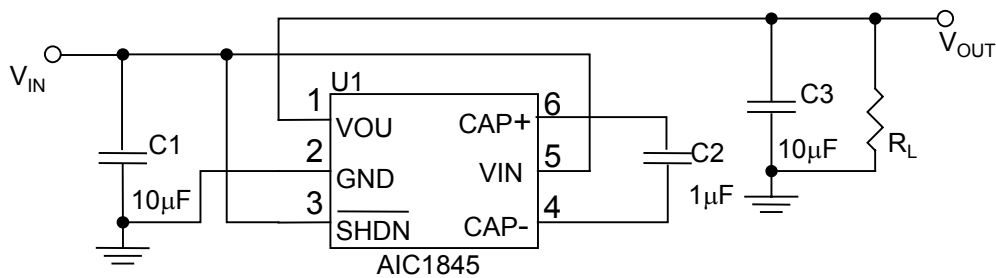


Fig. 5 The test circuit of AIC1845

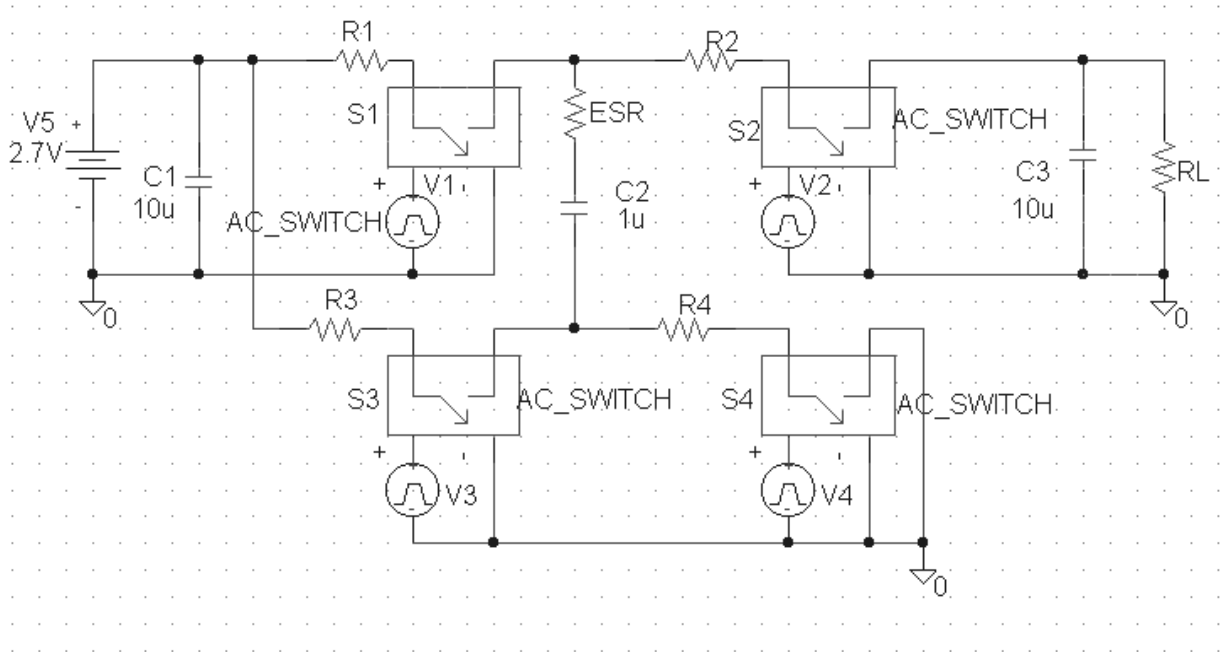


Fig.6 The simulation circuit of AIC1845

 Table. 1 The experiment results and the evaluated  $R_{DS-ON}$ .

$V_{IN}$ (V)	$R_{LOAD}$ ( $\Omega$ )	ESR (m $\Omega$ )	$V_{OUT}$ (V)	$I_{OUT}$ (mA)	Frequency(KHz)	$R_{DS-ON}$ ( $\Omega$ )
2.7	99.4	20	4.77	48.20	640	1.43
2.7	109.4	20	4.83	44.30	640	1.40
2.7	121.2	20	4.89	40.32	640	1.38
2.7	131.3	20	4.92	37.49	640	1.40
2.7	141.2	20	4.95	35.07	640	1.40

 Table. 2 The simulation results. ( $R_{DS-ON}$  is from Table.1)

$V_{IN}$ (V)	$R_{LOAD}$ ( $\Omega$ )	ESR(m $\Omega$ )	$R_{DS-ON}$ ( $\Omega$ )	Frequency (kHz)	$V_{OUT}$ (V)	$I_{OUT}$ (V)
2.7	99.4	20	1.43	640	4.83	48.62
2.7	109.4	20	1.40	640	4.89	44.68
2.7	121.2	20	1.38	640	4.94	40.77
2.7	131.3	20	1.40	640	4.97	37.84
2.7	141.2	20	1.40	640	5.00	35.38

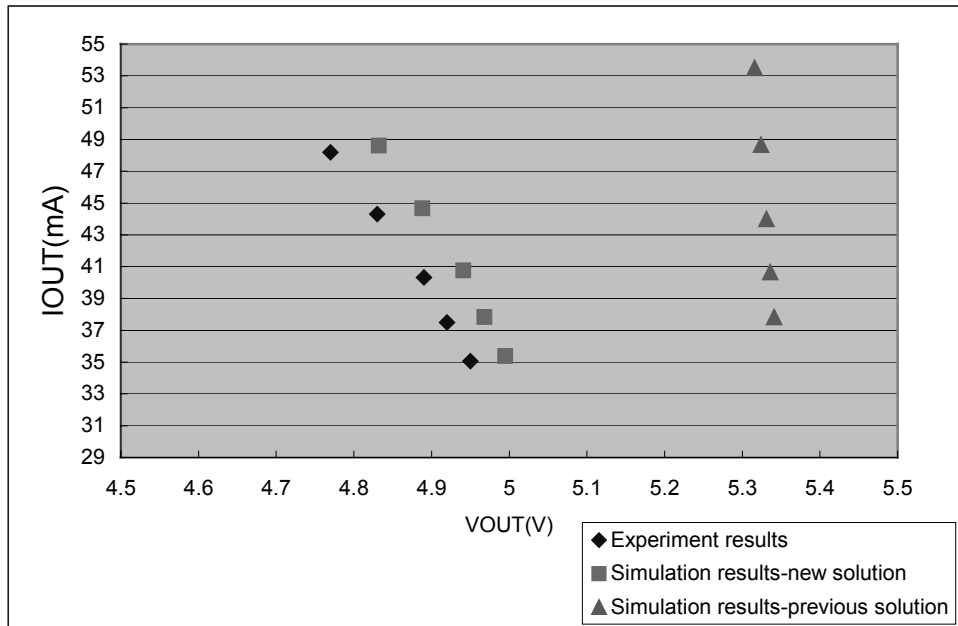


Fig. 7 The distribution of experiment results and simulation results

The previous solution means that the equivalent resistance includes only the  $1/f_C$  term.

**Summary:**

1. For the capacitive switched converter, the function of the flying capacitor  $C_1$  is storing and transferring charge. Due to the law of conservation of charge for capacitor, we can only obtain the relationship between input current and output current. Contrarily, in the inductor-used application, owing to the voltage-second balance of inductor, various voltage conversions are easily obtained. Therefore for the capacitor switched converter, to have an arbitrary voltage conversion is more difficult than inductor-used converter.

2. From the equivalent circuit shown in Fig.4, it is seen

that the terms  $1/fC_1$ ,  $4ESR$  and  $8R_{DS-ON}$  should be as small as possible to get large output current. However, for users, since the  $R_{DS-ON}$  is fixed and manufactured in IC, what we can do is to lower  $1/fC_1$  and ESR. However even the effect of  $1/fC_1$  and ESR can be kept as small as possible, the term  $8R_{DS-ON}$  still dominates the role that limits the maximum output current.

From Fig.4, the equivalent circuit shows a one-pole system. Therefore there is no need to worry about oscillation problem for charge pump converter.